

PART : MATHEMATICS

1. Let $y = f(x)$ is a solution of differential equation $e^y \left(\frac{dy}{dx} - 1 \right) = e^x$ and $f(0) = 0$ then $f(1)$ is equal to :

- (1) $\ln 2$ (2) $2 + \ln 2$ (3) $1 + \ln 2$ (4) $3 + \ln 2$

Ans. (3)

Sol. $e^y = t$

$$e^y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - t = e^x$$

$$IF = e^{\int -1 \cdot dx} = e^{-x}$$

$$t(e^{-x}) = \int e^x \cdot e^{-x} dx$$

$$e^{y-x} = x + c$$

Put $x = 0, y = 0$ then $c = 1$

$$e^{y-x} = x + 1$$

$$y = x + \ln(x + 1)$$

at $x = 1, y = 1 + \ln(2)$

2. If α is a roots of equation $x^2 + x + 1 = 0$ and $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$ then A^{31} equal to :

- (1) A (2) A^2 (3) A^3 (4) A^4

Ans. (3)

Sol. $A^2 = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$\Rightarrow A^4 = I$$

$$\Rightarrow A^{30} = A^{28} \times A^3 = A^3$$

3. The six digit numbers that can be formed using digits 1, 3, 5, 7, 9 such that each digit is used at least once.

Ans. 1800

Sol. 1, 3, 5, 7, 9

For digit to repeat we have 5C_1 choice

And six digits can be arrange in $\frac{6!}{2!}$ ways.






$$\text{Hence total such numbers} = \frac{5! \cdot 6!}{2!}$$

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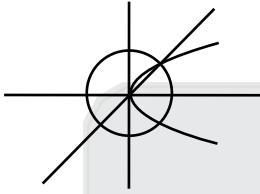
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4. The area that is enclosed in the circle $x^2 + y^2 = 2$ which is not common area enclosed by $y = x$ & $y^2 = x$ is
- (1) $\frac{1}{12}(24\pi - 1)$ (2) $\frac{1}{6}(12\pi - 1)$ (3) $\frac{1}{12}(6\pi - 1)$ (4) $\frac{1}{12}(12\pi - 1)$

Ans. (2)

Sol. Total area – enclosed area



$$2\pi - \int_0^1 \sqrt{x} - x \, dx$$

$$2\pi - \left(\frac{2x^{3/2}}{3} - \frac{x^2}{2} \right)_0^1$$

$$2\pi - \left(\frac{2}{3} - \frac{1}{2} \right) \Rightarrow 2\pi - \left(\frac{1}{6} \right) \Rightarrow \frac{12\pi - 1}{6}$$

5. If sum of all the coefficient of even powers in $(1 - x + x^2 - x^3 + \dots + x^{2n}) (1 + x + x^2 + x^3 + \dots + x^{2n})$ is 61 then n is equal to

- (1) 30 (2) 32 (3) 28 (4) 36

Ans. (1)

Sol. Let $(1 - x + x^2 + \dots) (1 + x + x^2 + \dots) = a_0 + a_1 x + a_2 x^2 + \dots$

put $x = 1$

$$1(2n+1) = a_0 + a_1 + a_2 + \dots + a_{2n} \quad \dots\dots(i)$$

put $x = -1$

$$(2n+1) \times 1 = a_0 - a_1 + a_2 + \dots + a_{2n} \quad \dots\dots(ii)$$

Form (i) + (ii)

$$4n + 2 = 2(a_0 + a_2 + \dots) = 2 \times 61 \quad \Rightarrow 2n+1 = 61 \Rightarrow n = 30$$

6. If variance of first n natural numbers is 10 and variance of first m even natural numbers is 16 then the value of m + n is

Ans. 18

Sol. $\text{Var}(1, 2, \dots, n) = 10 \Rightarrow \frac{1^2 + 2^2 + \dots + n^2}{n} - \left(\frac{1+2+\dots+n}{n} \right)^2 = 10$

$$\Rightarrow \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2} \right)^2 = 10$$

$$\Rightarrow n^2 - 1 = 120 \quad \Rightarrow n = 11$$

$$\text{Var}(2, 4, 6, \dots, 2m) = 16 \Rightarrow \text{var}(1, 2, \dots, m) = 4$$

$$\Rightarrow m^2 - 1 = 48 \quad \Rightarrow m = 7 \Rightarrow m + n = 18$$

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7. Evaluate $\lim_{x \rightarrow 2} \frac{3^x + 3^{x-1} - 12}{3^{\frac{-x}{2}} - 3^{1-x}}$.

Ans. 72

Sol. Put $3^{\frac{x}{2}} = t$

$$\Rightarrow \lim_{t \rightarrow 3} \frac{\frac{4t^2}{3} - 12}{\frac{1}{t^2} + \frac{1}{t}} = \lim_{t \rightarrow 3} \frac{4(t^2 - 9)t^2}{3(-3 + t)} = \lim_{t \rightarrow 3} \frac{4t^2(3 + t)}{3} = \frac{4 \times 9 \times 6}{3} = 72$$

8. If $f(x)$ is continuous and differentiable in $x \in [-7, 0]$ and $f'(x) \leq 2 \forall x \in [-7, 0]$, also $f(-7) = -3$ then range of $f(-1) + f(0)$

- (1) $[-5, -7]$ (2) $(-\infty, 6]$ (3) $(-\infty, 20]$ (4) $[-5, 3]$

Ans. (3)

Sol. Lets use LMVT for $x \in [-7, -1]$

$$\frac{f(-1) - f(-7)}{(-1 + 7)} \leq 2$$

$$\frac{f(-1) + 3}{6} \leq 2 \Rightarrow f(-1) \leq 9$$

Also use LMVT for $x \in [-7, 0]$

$$\frac{f(0) - f(-7)}{(0 + 7)} \leq 2$$

$$\frac{f(0) + 3}{7} \leq 2 \Rightarrow f(0) \leq 11 \quad \therefore f(0) + f(-1) \leq 20$$

9. If $y = mx + 4$ is common tangent to parabolas $y^2 = 4x$ and $x^2 = 2by$. Then value of b is

- (1) -64 (2) -32 (3) -128 (4) 16

Ans. (3)

Sol. $y = mx + 4$ (i)

$$y^2 = 4x \text{ tangent } y = mx + \frac{a}{m} \Rightarrow y = mx + \frac{1}{m} \text{(ii)}$$

from (i) and (ii)

$$4 = \frac{1}{m} \Rightarrow m = \frac{1}{4}$$

So line $y = \frac{1}{4}x + 4$ is also tangent to parabola

$x^2 = 2by$, so solve and $D = 0$

$$x^2 = 2b \left(\frac{x + 16}{4} \right)$$

$$\Rightarrow 2x^2 - bx - 16b = 0 \quad \Rightarrow D = 0 \Rightarrow b^2 - 4 \times 2 \times (-16b) = 0$$

$$\Rightarrow b^2 + 32 \times 4b = 0$$






$$b = -128, b = 0 \text{ (not possible)}$$

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10. If α and β are the roots of equation $(k+1) \tan^2 x - \sqrt{2} \lambda \tan x = 1 - k$ and $\tan^2 (\alpha+\beta) = 50$. Find value of λ .
- (1) 10 (2) 5 (3) 7 (4) 12

Ans. (1)

$$(k+1) \tan^2 x - \sqrt{2} \lambda \tan x + (k-1) = 0$$

$$\tan \alpha + \tan \beta = \frac{\sqrt{2} \lambda}{k+1}$$

$$\tan \alpha \tan \beta = \frac{k-1}{k+1}$$

$$\tan (\alpha+\beta) = (k-1) = 0 \Rightarrow \frac{\frac{\sqrt{2} \lambda}{k+1}}{1 - \frac{k-1}{k+1}} = \frac{\sqrt{2} \lambda}{2} = \frac{\lambda}{\sqrt{2}}$$

$$\tan^2 (\alpha+\beta) = \frac{\lambda^2}{2} = 50$$

$$\lambda = 10$$

11. Find image of point (2, 1, 6) in the plane containing points (2, 1, 0), (6, 3, 3) and (5, 2, 2)
- (1) (6, 5, -2) (2) (6, -5, 2) (3) (2, -3, 4) (4) (2, -5, 6)

Ans. (1)

Sol. Plane is $x + y - 2z = 3 \Rightarrow \frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = \frac{-2(2+1-12-3)}{6} \Rightarrow (x, y, z) = (6, 5, -2)$

12. Let $(x)^k + (y)^k = (a)^k$ where $a, k > 0$ and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then find k

(1) $\frac{1}{3}$

(2) $\frac{2}{3}$

(3) $\frac{4}{3}$

(4) 2

Ans. (2)

Sol. $k \cdot x^{k-1} + k \cdot y^{k-1} \frac{dy}{dx} = 0$

$$\frac{dy}{dx} = -\left(\frac{x}{y}\right)^{k-1}$$

$$\frac{dy}{dx} + \left(\frac{x}{y}\right)^{k-1} = 0$$

$$k - 1 = -\frac{1}{3}$$






$$k = 1 - \frac{1}{3} = \frac{2}{3}$$

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13. If $g(x) = x^2 + x - 1$ and $g(f(x)) = 4x^2 - 10x + 5$, then find $f\left(\frac{5}{4}\right)$.

यदि $g(x) = x^2 + x - 1$ और $g(f(x)) = 4x^2 - 10x + 5$, तब $f\left(\frac{5}{4}\right)$ का मान है-

- (1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) $-\frac{1}{3}$ (4) $\frac{1}{3}$

Ans. (2)

Sol. $g(f(x)) = f^2(x) + f(x) - 1$

$$g\left(f\left(\frac{5}{4}\right)\right) = 4\left(\frac{5}{4}\right)^2 - 10 \cdot \frac{5}{4} + 5 = -\frac{5}{4}$$

$$g\left(f\left(\frac{5}{4}\right)\right) = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$$

$$-\frac{5}{4} = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$$

$$f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) + \frac{1}{4} = 0$$

$$\left(f\left(\frac{5}{4}\right) + \frac{1}{2}\right)^2 = 0$$

$$f\left(\frac{5}{4}\right) = -\frac{1}{2}$$

14. If $z = x + iy$ and real part $\left(\frac{z-1}{2z+i}\right) = 1$ then locus of z is

- (1) Straight line with slope 2 (2) Straight line with slope $-\frac{1}{2}$
 (3) circle with diameter $\frac{\sqrt{5}}{2}$ (4) circle with diameter $\frac{1}{2}$

Ans. (3)

Sol. $z = x + iy$

$$\left(\frac{z-1}{2z+i}\right) = \frac{(x-1)+iy}{2(x+iy)+i} = \frac{(x-1)+iy}{2x+(2y+1)i} \times \frac{2x-(2y+1)i}{2x-(2y+1)i}$$

$$\operatorname{Re}\left(\frac{z-1}{2z+i}\right) = \frac{2x(x-1)+y(2y+1)}{(2x)^2+(2y+1)^2} = 1$$

$$\Rightarrow 2x^2 + 2y^2 - 2x + y = 4x^2 + 4y^2 + 4y + 1 \quad \Rightarrow 2x^2 + 2y^2 + 2x + 3y + 1 = 0$$

$$\Rightarrow x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0 \quad \text{Circle with centre } \left(-\frac{1}{2}, -\frac{3}{4}\right)$$






$$r = \sqrt{\frac{1}{4} + \frac{9}{16} - \frac{1}{2}} = \sqrt{\frac{4+9-8}{16}} = \frac{\sqrt{5}}{4}$$

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15. If distance between the foci of an ellipse is 6 and distance between its directrices is 12, then length of its latus rectum is

(1) 4

(2) $3\sqrt{2}$

(3) 9

(4) $2\sqrt{2}$

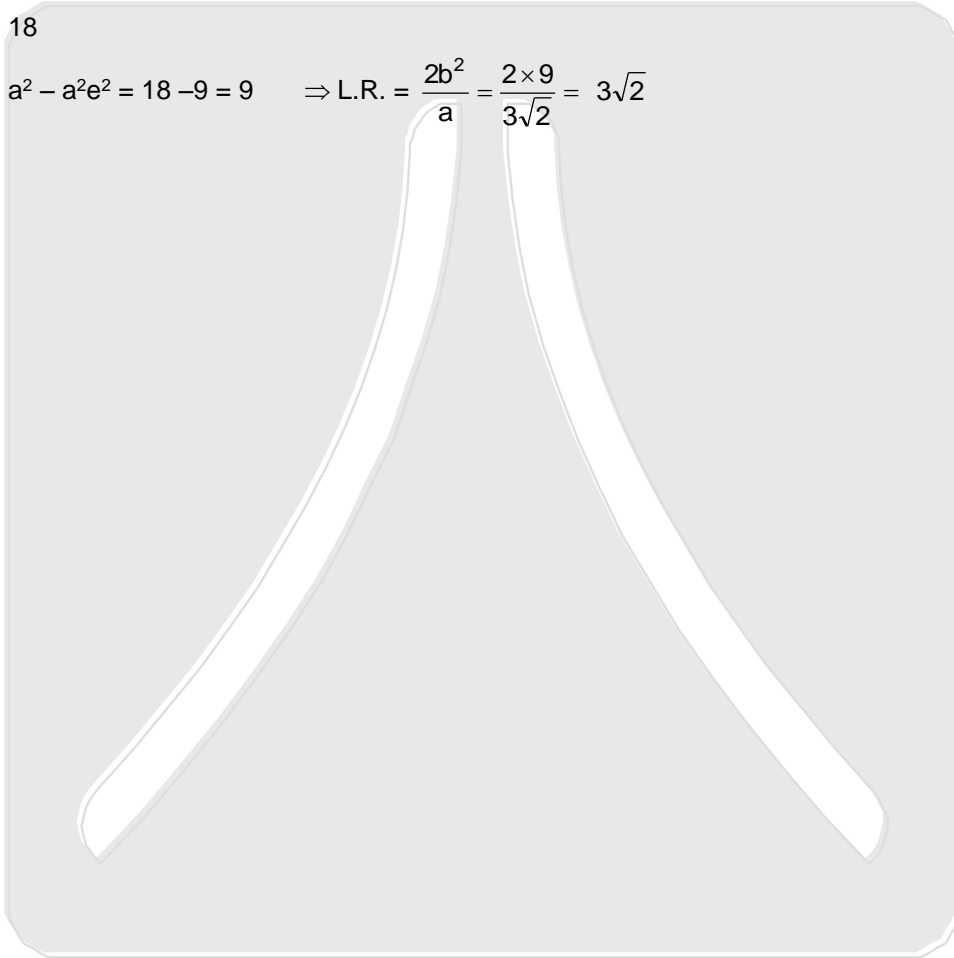
Ans. (2)

Sol. $2ae = 6$ and $\frac{2a}{e} = 12$

$\Rightarrow ae = 3$ and $\frac{a}{e} = 6$

$\Rightarrow a^2 = 18$

$\Rightarrow b^2 = a^2 - a^2e^2 = 18 - 9 = 9 \Rightarrow \text{L.R.} = \frac{2b^2}{a} = \frac{2 \times 9}{3\sqrt{2}} = 3\sqrt{2}$








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